1 Exercises 1

1. Pigeonhole Principle says: If you have more pigeons than pigeonholes, and each pigeon flies into some pigeonhole, then there must be at least one hole that has more than one pigeon.

   What happens, if you have as many pigeonholes as there are natural numbers, and as many pigeons, as there are integers? What about, if you have as many pigeons as there are natural numbers, but each pigeon tries to make nest with every other pigeon into a different hole? (Only one nest can be made into one hole.)

2. What is wrong in the following induction proof that all cats are of the same colour?

   Let n be the number of cats. If $n = 1$ the claim holds clearly (one cat is always of the same colour). Let's now suppose that for any group of n cats the claim holds. Then let's consider a group of $n + 1$ cats. By selecting any n cats from this group (which can be done in $n + 1$ different ways) we get by the induction assumption a group in which all the cats have the same colour. So all $n + 1$ cats must be of the same colour.

3. Let X be a set and X the size of $n = |X|$. Prove by induction that the size of the powerset of X is $|\mathcal{P}(X)| = 2^n$.

4. Prove the following claim. If there are $n(n \geq 2)$ people in the party, then at least two people have equal number of friends in the party.

5. Read the story about decision problems `http://www.cs.joensuu.fi/pages/whamalai/tfcs/story.html` and complete it!

6. Let's consider the logic school of Kissastan again. This time the topic is a more complicated MIAU-system, which consists of the following rules:

   xUAx → xAUy
   xUUx → xIUy
   x → MxM
   x → xUI
   xx → x

xI $\rightarrow$ xUA

The task is to show that even an empty string can create a proper miaow (MIAU) by the rules of the system!