Exercises about unsolvability

1. Show that the following semantic properties of recursive enumerable languages are nontrivial (trivial property holds for none or for all languages).

   a) L contains string w.
   b) L is finite.
   c) L is regular.
   d) L is $\{0,1\}^*$.

2. Give two examples of trivial properties: one, which doesn't hold for any recursive enumerable language and the other, which holds for all!

3. Which of the following properties of Turing machines are solvable? (Hint: either invent an algorithm idea or consider, if the property is semantic.)

   a) M halts on all even binary numbers.
   b) If M is started with empty input, it reaches state q on at most 10 steps, or if it is started with input a it reaches state p on at most 20 steps.
   c) M doesn't contain any transition into state q, in which it would write end character $<$.
   d) State q can be reached from state p on at most 3 steps.
   e) M has less than 100 states and M halts on input 0.

4. We know that for recursive reduction of problems $\leq_m$ holds:

   i) If $A \leq_m B$ and B is recursive enumerable, then A is recursive enumerable.

   i) If $A \leq_m B$ and B is recursive, then A is recursive.

   Is language X recursive, recursive enumerable or totally unsolvable in the following cases?

   a) For universal language U holds $U \leq_m X$.
   b) For language H and for an unknown recursive enumerable language Y holds $H \leq_m Y$ and $Y \leq_m X$. ($H = \{c_M w | M$ halts on input $w\}$)
   c) For H's complement $\overline{H}$ holds $\overline{H} \leq_m X$.
   d) For Hamiltonian Cycle -language $HC = \{x | x$ codes graph G, which contains Hamiltonian cycle$\}$ [1] holds $X \leq_m HC$.
   e) $HC \leq_m X$

[1] Hamiltonian cycle=cycle, which goes through all vertices exactly once.

N.B.! Remember the contraposition rule: $A \Rightarrow B \equiv \neg B \Rightarrow \neg A$.

5. Prove that the following property is unsolvable: Given a code of a Turing machine c_M, state q and input w. Does M enter by input w into state q?

6. Are the following languages recursive, recursive enumerable or totally unsolvable?

   a) $L_1 = \{c_M | \text{The code of machine } M \ c_M \text{ is palindrom}\}$.
   b) $L_2 = \{c_M | c_M \text{ is code of machine } M \text{ and } M \text{ recognizes all palindroms in alphabet } \{0,1\}\}$.

   (Definition for palindroms: Mark $w = a_0 a_1 ... a_n$, if $a_0 a_1 ... a_n = a_n a_{n-1} ... a_0$, then w is palindrom.)

7. Are the following problems solvable, partially solvable or totally unsolvable?

   a) Does the given Turing machine halt on all input?
   b) Does the given Turing machine halt on no input?
   c) Does the given Turing machine halt on at least one input?
   c) Does the given Turing machine fail to halt on at least one input?

8. The following problems are known to be unsolvable, but are they partially solvable or totally unsolvable?

   a) Does the language $L(M)$ contain at least two strings?
   b) Is $L(M)$ finite?
   c) Is $L(M)$ context-free language?

9. Invent some concrete examples of unsolvable problems! (Other than halting problem.) N.B.! Justify the unsolvability. Is the problem partially solvable (i.e. recursive enumerable language) or totally unsolvable problem?

10. Are the following problems solvable or unsolvable? If they are unsolvable, tell if they are partially solvable or totally unsolvable. If they are solvable, give the weakest type of machine (finite automaton, pushdown automaton, Turing machine), which solves the problem!

    a) Give an integer solution of x for arbitrary polynom $a_1 x^n + a_2 x^{n-1} + ... + a_1 x + a_0$!

b) Program for compiler an a error checker, which gets as its input a code of a program and checks that the program doesn't perform division by 0 during it execution.

c) Seach if the given text file contains either words **cat** and **black** or words **bird** and **white**, but not both.

d) Check that in the given text file word "animal" is followed by word "wise", only if word "cat" appears in the same sentence.

e) Construct a general virus tester, which recognizes all "dangerous" programs i.e. such programs, which can change system files during their execution.

11. Answer the course evaluation query! The course evaluation form can be found in `http://cs.joensuu.fi/ arvio/english.html`. Tell if you participated problem-based learning and how you felt it! Tell also, if you didn't take the problem-based way, and why it didn't suit for you! Thank you for your feedback!