Exercises about Pumping Lemma and context-free grammars

1. Are the following languages regular? State the reasons for your answer.

   (a) $\{w|w$ is a character string of length 3 consisting of a's and b's$\}$

   (b) $\{ww|w \in \{a,b\}^*\}$

   (c) $\{w^*|$there are as many 1's as 0's in w

   (d) $\{w|$there are twice as many 0's as 1's in w$\}$

2. We know that in general the problem $REG(L)$ (i.e. is the given language L regular or not) is unsolvable. What is the reason for that? Could you still invent a program, which would help people to prove by the Pumping Lemma that the given language is nonregular?

3. The stronger version of the Pumping Lemma gives both sufficient and necessary conditions for the regularity of a language:

   Pumping Lemma 2: The language $A \in \Sigma^*$ is regular if and only if there is a constant $n \geq 1$ such that for all $x \in \Sigma^*$, if $|x| \geq n$ then there exists u, v, w such that $x = uvw$ and $|v| \geq 1$, and for all $i \geq 0$ and all $yin\Sigma^*$ $xy \in A$ if and only if $uv^iwy \in A$.

   Based on this, could you make a program, which decides for any given language A, if it is regular or not?

4. Prove with the help of the Pumping lemma that the following languages are not regular, and create a context-free grammar describing the language.

   (a) $\{a^nb^nc^k|n, k = 0, 1, ...\}$

   (b) $\{a^nb^kc^k|n, k = 0, 1, ...\}$

   (c) $\{a^nb^na^mb^m|n, k = 0, 1, ...\}$

5. The languages produced by the following context-free grammars are regular. Describe the languages with regular expressions.

   (a) $S \to AS \mid \epsilon$
   $A \to a \mid b$

(b) $S \to SSS \mid a \mid b$

(c) $S \to AB$
$A \to aAa \mid bAb \mid \epsilon$
$B \to aB \mid bB \mid \epsilon$

6. All words of the language of outerspace aliens follows the Blurbs normalform. Blurb is a Whoozit, followed by one or more Whatzit. Whoozit is letter 'x', which can be followed by any number of letters 'y' (also zero). Whatzit is letter 'q', which is followed by either letter 'z' or 'd', followed by Whoozit.

   Give the context-free grammar, which describes the Blurbs language. Can you now give the corresponding regular expression? (Hint: construct first the automaton from the grammar!)

7. For regular expression holds the following rule::
   If $r = rs \bigcup t$, then $r = ts^*$, when $\epsilon \notin L(s)$.
   Show, why this rule holds with the help of context-free grammars!

8. Create a grammar that is linear to the right and describes the language

   $\{w \in \{a, b\}^* | w$ does not contain character string $abaa\}$

9. Create a context-free grammar that produces programming language expressions consisting of an infinite number of 'for' loops inside each other and elementary operations a, in the same way as in the example.

```
for (i = N; i < N; i++) {
    for (j=N; j<N; j++) {
        a;
    }
}
```

   in which N is an integer.

10. Describe the following C-programs as context-free grammar!

```
for (i = N; i < N; i++) {
    for (j=N; j<N; j++) {
        a;
    }
}
```

in which N is integer.

- the function takes the form

  fcttype fctname(partype par (,partype par)*)
  {
    fcttrunk
  }

  in which fcttype may be void or int and partype is int.
- fcttrunk is an ifexpression or assignment
- the assignment takes the form variable=value;
- par and the variable are variable names consisting of one letter
- the value is an integer
- the ifexpression takes the form
  if (condition) ifexpression assignment; or
  if (condition) ifexpression assignment; else ifexpression assignment; or
  an empty character string
- the condition takes the form (variable=value)