Case 9

You are given a very simple computer, which consists of a central unit, three working tapes and three reading heads, each of which can read or write one character at once and move one step to the right (R) or left (L). The behavious of the central unit is described as a transition function of form $\delta(q, a_1, a_2, a_3) = (q', (b_1, \Delta_1), (b_2, \Delta_2), (b_3, \Delta_3))$ (i.e. in state q and reading from tapes 1,2 and 3 the characters a_1, a_2 and a_3, the machine transfers to the state q', writes into tapes characters b_1, b_2 and b_3, and moves the reading heads into directions Δ_1, Δ_2 and Δ_3, in which $\Delta_i = \{L, R\}$.

Your task is to "program" with the given machine a summing calculator, which gets in the tapes 1 and 2 two integers in binary form and calcutes the sum of them into third tape. For simplicity you can suppose that the integers are represented the least significant bit in the left and if the integers are of different length, the shorter one is added extra zeros into end. For example the computation 7+11=18 would be represented as

1110
1101
01001

Your "program" should describe the corresponding transition function, e.g. as a transition diagram (code each actions associated to the transition in form $(a_1/b_1, \Delta_1), (a_2/b_2, \Delta_2), (a_3/b_3, \Delta_3)$).

Could you construct the equivalent calculator with a similar computer, which has only one working tape?