Exercise session 6

1. Draw the following pushdown automaton M:

   $M = (\{q_0, q_1, q_2\}, \{a, b\}, \{A\}, \delta, q_0, \{q_1, q_2\})$, in which
   $\delta(q_0, a, \epsilon) = \{(q_0, A)\}$
   $\delta(q_0, \epsilon, \epsilon) = \{(q_1, \epsilon)\}$
   $\delta(q_0, b, A) = \{(q_2, \epsilon)\}$
   $\delta(q_1, \epsilon, A) = \{(q_1, \epsilon)\}$
   $\delta(q_2, b, A) = \{(q_2, \epsilon)\}$
   $\delta(q_2, \epsilon, A) = \{(q_2, \epsilon)\}$

   What kind of language does M recognize? Trace all computation paths with input strings *aab*, *abb* and *aba*! Does M accept strings *aabb* and *aaab*?

2. Write that w^R = the character string w backwards (i.e. if $w = a_1 a_2 \dots a_n$, then $w^R = a_n \dots a_2 a_1$). The character string is a palindrome if $w = w^R$ (e.g. "madamimadam"). Consider the language PAL $= \{ w \in \{ a, b \} \mid w = w^R \}$. that is formed by palindromes in the alphabet $\{a, b\}$.

   a) Give a context-free grammar, which produces the language.
   b) Construct a pushdown automaton, which recognizes the language.

3. Create a context-free grammar that produces Roman numbers that are less than 90. The numbers are L=50, X=10, V=5, I=1. XII=12, for example, and XIV=14, XL=40, XLIX=49. For thew sake of simplicity, you may assume that an empty character string signifies 0 and the character string LXL the number 90.

4. Construct a pushdown automaton, which recognizes the Roman numbers that are less than 90, as described in the previous task!

5. In the Logic school of Kissastan the current topic is context-free languages. In this class the cats consider the following language:

   One miaow can begin with one or more *miu* and in the end there is equal number of *mau*'s. Between them can be 0 or more *u*'s. Or the miaow is maow, which is followed by one or more *au*. Maow begins with *miau* or *mau*, which is followed by 0 or more *u*'s.

a) Give a context-free grammar, which produces the language.
b) Give some examples words of this language.
c) Construct a pushdown automaton, which recognizes this language.

6. Show that the following grammars are ambiguous! Can you give an unambiguous grammar which describes the same language?

   (a) $S \to aSb|SS|\epsilon$

   (b) $S \to aSb|aaSb|\epsilon$

7. Construct a grammar which produces language $\{a^m b^n c^{m+n} | m, n \geq 0\}$. Is your grammar ambiguous or unambiguous?

8. What does it matter if a language is inherently ambiguous?

9. Show that the following languages are context-free!:

   a)$\{a^m b^n | m \geq n\}$
   b)$\{a^m b^n c^p d^q | m + n = p + q\}$
   c)$\{uawb | u, w \in \{a, b\}^*, |u| = |w|\}$
   d)$\{a^m b^n | n \leq m \leq 2n\}$

10. Let $\Sigma = \{m, i, u, a\}$. Let's consider the grammar:
    $S \to miuNmauS|SmiauA|\epsilon$
    $N \to miuNmau|U$
    $U \to uU|\epsilon$
    $A \to Aau|\epsilon$

    Give the leftmost and the rightmost derivation and the parsing tree for the string
    $miumiuuumaumaumiaumiauau$! Is the grammar unambiguous or ambiguous?

More challenging

11. What kind of language does the following pushdown automaton M recognize?
    $M = (\{q_0, q_1\}, \{a, b\}, \{\underline{A}, \underline{A_1}, \underline{B}, \underline{B_1}, A, A_1, B, B_1\}, \delta, q_0, \{q_0\})$

$\delta(q_0, a, \epsilon) = \{(q_1, \underline{A_1})\}$
$\delta(q_0, b, \epsilon) = \{(q_1, \underline{B})\}$
$\delta(q_1, a, \epsilon) = \{(q_1, \underline{A_1})\}$
$\delta(q_1, a, B) = \{(q_1, B_1)\}$
$\delta(q_1, a, B_1) = \{(q_1, \epsilon)\}$
$\delta(q_1, a, \underline{B}) = \{(q_1, \underline{B_1})\}$
$\delta(q_1, a, A_1) = \{(q_1, A)\}$
$\delta(q_1, a, \underline{A_1}) = \{(q_1, \underline{A})\}$
$\delta(q_1, b, \epsilon) = \{(q_1, B)\}$
$\delta(q_1, b, A) = \{(q_1, \epsilon)\}$
$\delta(q_1, b, A_1) = \{(q_1, B_1)\}$
$\delta(q_1, b, \underline{A_1}) = \{(q_1, \underline{B_1})\}$

12. Create a context-free grammar, which describes the polynoms of a variable x. For the sake of simplicity, you may assume that the coefficients and the exponents of the terms are integers of one digit and that the first term is without prefix. The terms do not have to be in a specific order, and there may be temrs of the same denomination in the polynom. Give the derivation trees of the following strings in your grammar: $2 * x\hat{}2 - 2 * x + 1$, $x - x\hat{}2 - 1 + 2 * x\hat{}2 - 3x$, x, 5! Can you construct a recursive parser for the language?

13. Give some middle term feedback about the course! The course evaluation form can be found in `http://cs.joensuu.fi/ arvio/english.html`. Mention somewhere, if you take part in problem-based learning or perform the course in the traditional way. If you have chosen the problem-based way, tell if you feel learning in that way easier/harder than traditional learning. If you have chosen the traditional way, tell why the problem-based learning didn't suit for you. Remember to mention the name of the teacher you are refering! Thank you for your feedback!