Exercise session 9

1. Give an unrestricted (or context-sensitive) grammar, which produces language $L = \{a^i b^{2i} a^i | i > 0\}$. Give the derivation for string *aabbbbaa* in the grammar!

2. Chomsky thought that natural language can be described by context-sensitive grammars, in which all productions can be represented in form:

   $S \to \epsilon$ or $\alpha A \beta \to \alpha \omega \beta$,

   in which A is non-terminal symbol and $\omega \neq \epsilon$. Give some example of (syntactic) structure in natural language, which requires such "context" α_β! (You can consider examples in your own native language, but explain the feature also in English.) Do you believe that all structures of natural language can be described with context-sensitive grammars?

3. The Universal Turing machine has invited all Turing machines of the universum into Universal Truring Symposium, which is hold in Hilbert's Hotel. The room reservations have been done with the codes c_M of the Turing machines. Is there enough rooms for all Turing machines, if there are no other guests in the hotel? How would you allocate the rooms?

4. Draw the Turing machine described by the following code!

   111010100101001101001010010011010000100001000010110010101010011
   00100100100100110010000100010000 10111.

5. What are the codes of the following Turing machines?

   (a) $\delta(q_0, 0) = (q_0, 0, R)$
   $\delta(q_0, 1) = (q_0, 1, R)$
   $\delta(q_0, <) = (q_{yes}, <, L)$

   (b) $\delta(q_0, 0) = (q_1, 0, R)$
   $\delta(q_0, 1) = (q_{no}, 1, R)$
   $\delta(q_0, <) = (q_{no}, <, L)$
   $\delta(q_1, 0) = (q_{yes}, 0, R)$
   $\delta(q_1, 1) = (q_0, 1, R)$
   $\delta(q_1, <) = (q_{yes}, <, L)$

6. Simulate the behaviour of the universal Turing machine, when it is given as its input $c_M w$ the code of the machine in b-part of the previous task and a string 0101!

7. Construct a finite automaton, which recognizes the legal codes of Turing machines. What is the corresponding regular expression?

8. Construct a standard Turing machine, which recognizes the legal codes of Turing machines.

9. Let the languages A and B be recursive enumerable. (i.e. there exists Turing machines M_A and M_B, which recognize the given languages and halt in "Yes"-case, but not necessarily in "No"-case.) Prove that languages $A \cup B$ and $A \cap B$ are also recursive enumerable i.e. you can construct for them Turing machines, which halt at least in "Yes"-case for all input strings!

10. Let A be a recursive enumerable language. Can you construct a Turing machine for its complement language $\overline{A}$ from the machine M_A by changing the accepting and rejecting final states?

11. Prove that language $\{c_M | M \text{ halts on input } \epsilon\}$ is recursive enumerable but not recursive! I.e. invent for it a Turing machine, which halts in "Yes"-case, but not necessarily in "No"-case!

12. Give an example of a Turing machine, which

    a) accepts its own code.
    b) does not accept its own code.

13. Show that the following semantic properties of recursive enumerable languages are nontrivial (trivial property holds for none or for all languages).

    a) L contains string w.
    b) L is finite.
    c) L is regular.
    d) L is $\{0, 1\}^*$.

14. Give two examples of trivial properties: one, which doesn't hold for any recursive enumerable language and the other, which holds for all!

15. Which of the following properties of Turing machines are solvable? (Hint: either invent an algorithm idea or consider, if the property is semantic.)

    a) M halts on all even binary numbers.
    b) If M is started with empty input, it reaches state q on at most 10 steps, or if it is started with input a it reaches state p on at most 20 steps.
    c) M doesn't contain any transition into state q, in which it would write end character $<$.
    d) State q can be reached from state p on at most 3 steps.
    e) M has less than 100 states and M halts on input 0.

16. Prove that the following property is unsolvable: Given a code of a Turing machine c_M, state q and input w. Does M enter by input w into state q?

17. Are the following languages recursive, recursive enumerable or totally unsolvable?

    a) $L_1 = \{c_M | \text{The code of machine } M\ c_M \text{ is palindrom}\}$.
    b) $L_2 = \{c_M | c_M \text{ is code of machine } M \text{ and } M \text{ recognizes all palindroms in alphabet } \{0,1\}\}$.

    (Definition of palindroms: Denote $w = a_0a_1...a_n$, if $a_0a_1...a_n = a_na_{n-1}...a_0$, then w is palindrom.)